

# **MIMO Radar Aperture Optimization**

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**MIMO Radar Aperture Optimization**

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## ABSTRACT

In a multiple-input, multiple-output (MIMO) radar system, two or more transmitters emit independent waveforms, with the resulting reflections received by an array of receivers. Recently, MIMO radar has become a subject of great interest. In part, this interest is due to the potential for MIMO techniques to reduce radar weight and cost, while maintaining performance (as compared with conventional radar approaches). However, the size of these reductions has not yet been quantified. Likewise, a design process that minimizes aperture cost (or weight) has yet to be developed.

This report describes a process for designing optimal radar apertures. The process treats the design problem as one of minimizing an objective function under performance constraints. The objective function is based upon a first-order model for the relationship between cost (or weight) and performance, and is derived for systems employing active, element-digitized arrays. A systematic process for optimizing the aperture's design with respect to this objective function is presented, and equations describing the optimal aperture are derived. These equations provide insight into the optimal relationship between various aperture characteristics, such as the number of transmitters, number of receivers, module power level, and virtual array length.

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## 1. INTRODUCTION

In recent years, Multiple-Input, Multiple-Output (MIMO) radars have attracted great interest [1–15]. In large part, this interest stems from the potential for MIMO techniques to yield lighter-weight, lower-cost airborne and surface-based radar systems (as compared with conventional phased arrays of comparable performance). Other benefits include increased angle [1–6] and Doppler [1,7] resolution, reduced clutter levels [1,8], sharper moving target indication (MTI) clutter notches (yielding lower minimum detectable velocity [MDV]) [2,7], lower probability of intercept [1,8], and relaxed hardware requirements [1,8]—when compared with conventional radars of similar complexity.

These benefits all stem from the unique fashion in which MIMO radars transmit, receive, and process signals. In MIMO radar,  $N_t$  antennas each transmit a unique waveform. These waveforms propagate through the environment, where they are reflected by targets and other objects, and subsequently received by  $N_r$  digital receivers. Each receiver's output is then processed by a bank of  $N_t$  matched filters (i.e., one matched to each of the transmitted waveforms). As such, each filter essentially recovers the component of the received signal due to a single transmitter. Moreover, the phase observed at the output of each filter behaves like that of a signal received by virtual sensor, i.e., by a phantom antenna element, located at the effective phase center of its constituent transmitter/receiver pair. Better yet, since there are a total of  $N_t \cdot N_r$  matched filters each producing data, there will be a total of  $N_t \cdot N_r$  output signals, each acting like it originates from a phantom element positioned within some  $N_t \cdot N_r$  element “virtual antenna array.”

As with any conventional phased array, the  $N_t \cdot N_r$  channels that form the MIMO virtual antenna array can be combined as desired, e.g., to form beams or maximize signal-to-interference plus noise ratio (SINR). Not surprisingly, the performance achieved by combining such channels will depend strongly upon array topology. In the case of MIMO radars, in particular, it is the *virtual* array's topology that is paramount. This topology depends directly upon the types of apertures used for transmitting and receiving signals.

To date, most coherent MIMO radar concepts have employed one of two apertures types. In the first type, waveforms are transmitted and received using a fully filled array, as shown in Figure 1(a) (top). In this case, the  $N_t \cdot N_r$  channels form a virtual array that is densely packed, with a total length that is nearly twice as long as the original T/R array. This type of virtual array, which is depicted in Figure 1(b) (top), provides the flexibility to operate in both MIMO and single-input multiple-output (SIMO) modes, while providing enhanced performance in MIMO search modes along with excellent MIMO antenna pattern control.

In contrast, radar applications that are driven by aperture length and spatial resolution (e.g., airborne Ground Moving Target Indicator [GMTI] radars) often prefer a second type of MIMO aperture. In this second type of system, waveforms are transmitted and received using a pair of real arrays, see Figure 1(a) (bottom). One of these arrays is large and highly sparse (i.e., undersampled spatially), while

the other is small and fully filled (i.e., Nyquist sampled spatially). This arrangement is attractive because, with appropriately spaced elements, the resulting virtual array will be both very long and fully filled, as shown in Figure 1(b) (bottom). Moreover, since the virtual aperture is uniformly filled, the resulting antenna patterns will be well behaved. As such, MIMO provides such systems with the benefits of a large aperture, with reduced weight and cost, and without the elevated sidelobe levels that are typical of sparse arrays.

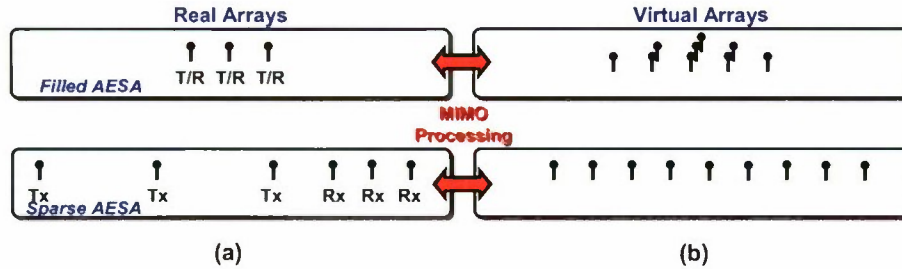


Figure 1. (a) Typical MIMO radar apertures. (b) Associated virtual arrays. Here, "T/R" denotes an integrated transmit/receive element, "Tx" denotes a transmit-only element, and "Rx" denotes a receive-only element.

To reduce the number of antennas used to cover a desired virtual aperture, this second type of MIMO aperture is preferred. In fact, it is easy to show that the number of antennas will be minimized by setting the number of transmit and receive elements to be equal. As such, this design choice minimizes "complexity." Some have even said that such systems will "minimize" cost (or weight). However, there is no simple relationship between complexity and cost (or weight), and thus no quantitative analysis has been performed to justify this conclusion. Furthermore, no radar system design procedure has been given for optimizing the design of such arrays with respect to cost or weight.

To address this problem, we outline a process for optimally choosing the number of transmit and receive elements, virtual aperture length, and transmitted power level, under constraints on search rate and detection performance. This report considers both conventional (SIMO) phased array radar systems employing filled apertures and MIMO radars employing sparse apertures. It is shown that the "typical" sparse MIMO design approach described in the literature (i.e., using equal number of transmit and receive elements) is usually suboptimal. Furthermore, we quantify the costs associated with optimal MIMO and SIMO systems, and show that a somewhat modest aperture cost savings is possible by using an optimally designed MIMO aperture, with larger savings achieved when such systems are also constrained to provide "excess spatial resolution." These cost savings will be offset, to some degree, by the increased processing load required for MIMO systems. Processing costs are not addressed in this report, but may be considered at a future time.



## 2. MIMO RADAR PRINCIPLES

Consider an array of  $N_t$  transmit antennas and  $N_r$  receive antennas (the optimal configuration of these antennas is discussed in Section 3). In MIMO radars, each of the transmit antennas radiates an independent waveform. Thus, we let  $\mathbf{s}_n$  denote the signal radiated by transmit antenna  $n$ , where  $n=1, \dots, N_t$  (i.e.,  $\mathbf{s}_n$  is a  $P_w \times 1$  complex vector containing baseband samples of the  $n^{\text{th}}$  radiated signal). Furthermore, let  $\theta_t$  represent various parameters (e.g., location coordinates) associated with target  $t$ , and let  $\mathbf{a}_{Tx}(\theta_t)$  be the corresponding  $N_t \times 1$  response vector for the transmit antenna array. Under these conditions, the signal reflected from point-target  $t$  is proportional to  $\mathbf{a}_{Tx}^T(\theta_t)\mathbf{S}$ , where the  $N_t \times P_w$  signal matrix  $\mathbf{S}$  is defined via  $\mathbf{S} = [\mathbf{s}_1 \ \mathbf{s}_2 \ \dots \ \mathbf{s}_{N_t}]^T$ .

The reflected signal propagates through the environment and is received by an array of antennas. Assuming each receiver collects  $R \gg P_w$  samples, the  $N_r \times R$  sample matrix at the output of the receive array has the form

$$\mathbf{X} = \sum_t \alpha(\theta_t) \mathbf{a}_{Rx}(\theta_t) \mathbf{a}_{Tx}^T(\theta_t) \mathbf{S}_t + \sum_c \alpha(\theta_c) \mathbf{a}_{Rx}(\theta_c) \mathbf{a}_{Tx}^T(\theta_c) \mathbf{S}_c + \mathbf{N}, \quad (1)$$

where subscripts  $t$  and  $c$  index into the set of targets and clutter patches, respectively. Here,  $\alpha(\theta_t)$  represents a complex scale factor associated with target  $t$ , while  $\mathbf{a}_{Rx}(\theta_t)$  denotes the receive array's  $N_r \times 1$  response vector for target  $t$ . The  $N_t \times R$  matrix  $\mathbf{S}_t$ , on the other hand, describes how the various waveforms propagate to, and are reflected by, target  $t$ . That is, the  $n^{\text{th}}$  column of  $\mathbf{S}_t$  contains the convolution of  $\mathbf{s}_n$  with the target's impulse response. Furthermore, since  $R \gg P_w$ ,  $\mathbf{S}_t$  will also contain many zeros to account for propagation delays to/from the target. Variables  $\alpha(\theta_c)$ ,  $\mathbf{a}_{Rx}(\theta_c)$ ,  $\mathbf{a}_{Tx}(\theta_c)$ , and  $\mathbf{S}_c$  are defined similarly with respect to each clutter patch. Lastly, the  $N_r \times R$  matrix  $\mathbf{N}$  represents noise as well as other interference.

After being received, the sample matrix is processed to extract target detections and/or to form images. Target detection, for example, can be performed by passing the received samples through a bank of matched filters. Often, this filtering is implemented in stages, wherein the initial stage applies a set of "waveform" matched filters to form  $N_t \times R'$  matrices,

$$\mathbf{Y}_m = \begin{bmatrix} \mathbf{x}_m^T * \mathbf{h}_1^T \\ \vdots \\ \mathbf{x}_m^T * \mathbf{h}_{N_t}^T \end{bmatrix}, \quad m=1, \dots, N_r, \quad (2)$$

where  $\mathbf{x}_m^T$  denotes the  $m^{\text{th}}$  row of  $\mathbf{X}$ ,  $*$  denotes convolution, and  $\mathbf{h}_n$  denotes the matched filter for signal  $\mathbf{s}_n$ . After matched filtering, the rows of the  $N_t N_r \times R'$  matrix  $\mathbf{Y} = [\mathbf{Y}_1^T \ \dots \ \mathbf{Y}_{N_r}^T]^T$  represent  $N_t N_r$  channels of data. It is thus possible to linearly combine these channels,

$$\mathbf{z} = \mathbf{w}^H \mathbf{Y} , \quad (3)$$

to achieve some desired goal, such as the formation of beams [1,3,8] or the maximization of SINR [2,3].



### 3. SEARCH RADAR APERTURE OPTIMIZATION

In this section, the problem of designing optimal search radar apertures is addressed. Optimization is performed from a cost perspective, under performance constraints. Comparisons between optimized MIMO and SIMO systems are presented.

#### 3.1 OPTIMIZED MIMO SEARCH RADAR

To begin, let us assume the use of a critically sampled MIMO virtual aperture of length  $L$ . Assuming each element has physical aperture length of  $\lambda/2$ ,  $N_t$  is given by

$$N_t = 2L/(\lambda \cdot N_r) . \quad (4)$$

Next, impose constraints upon the search performance of this system by requiring it to have a predefined power-aperture (PA) product. For MIMO systems, the PA product is proportional to  $\xi$ , where

$$\xi = P_m \cdot N_t \cdot N_r , \quad (5)$$

and  $P_m$  is the average RF power radiated from each transmit antenna. Substituting our expression for  $N_t$  into the PA product, we have

$$\xi = 2 \cdot L \cdot P_m / \lambda . \quad (6)$$

Assuming the length of the MIMO virtual aperture is a free variable (i.e., the resolution is unconstrained), we can solve for  $L$  as a function of  $\xi$ , yielding

$$L = \xi \lambda / (2 P_m) . \quad (7)$$

Our goal, then, is to choose  $N_r$ ,  $N_t$ , and  $P_m$  so as to minimize radar aperture cost under the performance constraint above. To accomplish this goal, we first define a model relating aperture design characteristics to cost. Table 1 lists various aperture components, and describes their contribution to cost. In particular, each row in Table I describes a specific component of the system. Within the row, columns describe how the component's cost is related to fundamental aperture characteristics. For example, consider the antenna receiver (Rx) module, which includes the low noise amplifier (LNA) and other devices. As  $N_r$  increases, the number of such modules must also increase (for both MIMO and SIMO systems), thereby raising their contribution to overall aperture cost proportionally. In Table 1, this relationship is shown by the entering " $N_r$ " in the columns labeled "Dependence on  $N_t$  or  $N_r$ ."

Similarly, other component costs (e.g., the low power transmit circuitry) will depend solely on a single aperture characteristic such as  $N_t$  or  $N_t P_m$  (i.e., total radiated power). However, not all component costs will depend on a single characteristic. Some costs will depend on multiple factors. For example, the cost associated with the high power portion of each transmit module (which includes the high-power amplifier and other circuitry) will add to the total aperture cost in a way that depends both on  $N_t$  and

$P_m/E_m$  (i.e., the power to each module). Likewise, the cost to cool each module will depend on three factors:  $N_t$ ,  $(1-E_m)P_m/E_m$  (i.e., the power dissipated by the module), and  $(1-E_m)P_m/(E_m \cdot L \cdot H)$  (i.e., the power density of the array, in watts dissipated per square meter, assuming an  $L \times H$  tile-based array).

For the sake of simplifying the analysis that follows, we assume (1) the use of a DC-to-DC converter at each transmit element, and (2) the exponent values ( $a$ ,  $b$ ,  $c$ , and  $d$ ) in Table 1 are equal to 1. These assumptions lead us to a closed form solution for the optimum array design, avoiding the need for numerical solutions. In practice, though, a more accurate model can be obtained by removing these assumptions.

**Table 1**  
**Cost Model for SIMO and MIMO Radar Apertures\***

SUBSYSTEM	COMPONENT	%	Fixed Term	Dependence on $N_t$ or $N_r$		Dependence on Power	Dependence on Power Density
				SIMO Model	MIMO Model	(times $N_t$ )	(times $N_t$ )
Antenna	Radiator			$N_t$	$N_t, N_r$	--	--
	Structure		Yes	$N_t$	$(N_t, N_r)$	--	--
	High Power Tx			$N_t$	$N_t$	$(P_m/E_m)^a$	--
	Low Power Tx			$N_t$	$N_t$	--	--
	Rx			$N_r$	$N_r$	--	--
	DC-DC Conversion			Variable	Variable	$(P_m/E_m)^b$	--
	Module Cooling			$N_t$	$N_t$	$(1-E_m)^c P_m/E_m$	$(1-E_m)P_m/(E_m \cdot L \cdot H)$
Receiver / Exciter	Centralized Exciter		Yes	--	--	--	--
	Distributed Exciter			$N_t$	$N_t$	--	--
	Distributed Receiver			$N_r$	$N_r$	--	--
Power & Cooling	Power Conditioning			--	--	$(P_m/E_s)^c$	--
	Chilling			--	--	$(P_m(1-E_s)/E_s)^d$	--
Overhead	Test & Evaluation	Yes		--	--	--	--
	Management	Yes		--	--	--	--

\*Where  $E_m$  denotes module efficiency and  $E_s$  is system efficiency. The model assumes: (1) active, tile-based transmit arrays and element-digitized receive arrays, (2) available DC power, (3) noncontiguous sparse antenna structure, and (4) module cooling costs that depend on power and/or power density.

Continuing under these assumptions, Table 1 suggests the following parametric model for aperture hardware cost:

$$(C_t + C_p P_m + C_d P_m / L) N_t + C_r N_r + C_f \quad (8)$$

The total cost to build and test the aperture, in contrast, is modeled as the sum of the array hardware cost (8) plus a percentage to cover test, evaluation, and management. Nonetheless, the best set of design parameters will be those that minimize (8).

Note that the coefficients in our aperture cost model (8) play an important role: they quantify relationships between various aperture design characteristics and aperture cost. The coefficient  $C_f$ , for

example, represents the total fixed cost (i.e., the portion of cost that is independent of the aperture size, power level, or power density). Likewise, the coefficients  $C_r$ ,  $C_t$ ,  $C_p$ , and  $C_d$  are used to describe the dependence of cost on  $N_r$ ,  $N_t$ ,  $N_t P_m$ , and  $N_t P_m / L$ , respectively (where  $L$  is the length of the active portion of the aperture; array height  $H$  is assumed to be fixed). The actual values assigned to these coefficients (i.e.,  $C_f$ ,  $C_r$ ,  $C_t$ ,  $C_p$ , and  $C_d$ ) will be, in practice, determined empirically (e.g., by itemizing the cost for each component in an existing radar system, then factoring these costs into the form given by (8)).

Furthermore, optimizing with respect to the parametric form in (8) allows us to investigate certain special cases directly. For example, we can set  $C_d = 0$  in (8), yielding a simpler cost model. Setting  $C_d = 0$  implies the cost to cool the aperture does not depend on power density. In contrast, allowing  $C_d$  to be nonzero allows one to investigate, to a degree, the impact of higher cost cooling technologies (e.g., liquid cooling versus forced air) needed for high power density radars. Note that (8) models the relationship between cost and power density as linear. In practice, this is an approximation as illustrated in Figure 2. Additional accuracy is possible by allowing costs associated with cooling to grow nonlinearly with power density. Such models, however, typically require numerical solution.

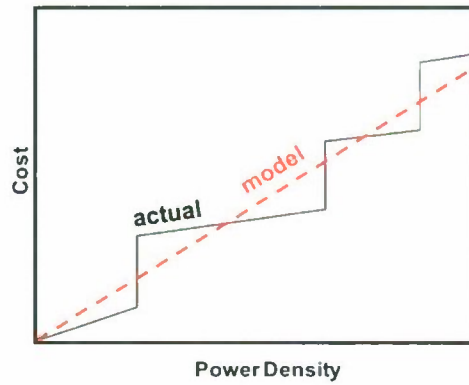


Figure 2. First order model relating cost to power density/cooling technology.

Substituting our expressions for  $N_t$  and  $L$  above, we may rewrite our cost function directly in terms of  $N_r$  and  $P_m$ :

$$\text{Cost}(N_r, P_m) = C_f + C_r N_r + \frac{2C_d P_m^2 + C_p \xi \lambda P_m + C_t \xi \lambda}{N_r P_m \lambda} \quad (9)$$

The partial derivatives with respect to  $N_r$  and  $P_m$  are

$$\frac{\partial}{\partial N_r} \text{Cost}(N_r, P_m) = C_r - \frac{2C_d P_m^2 + C_p \xi \lambda P_m + C_t \xi \lambda}{N_r^2 P_m \lambda} \quad (10)$$

and

$$\frac{\partial}{\partial P_m} \text{Cost}(N_r, P_m) = \frac{4C_d P_m + C_p \xi \lambda}{N_r P_m \lambda} - \frac{2C_d P_m^2 + C_p \xi \lambda P_m + C_t \xi \lambda}{N_r P_m^2 \lambda} . \quad (11)$$

To find the minimum cost system, we set these partial derivatives equal to zero and solve for  $N_r$  and  $P_m$  as a function of  $C_t$ ,  $C_p$ ,  $C_d$ , and  $\lambda$ . One solution is

$$N_r = \sqrt{\frac{\sqrt{8C_t C_d \xi \lambda} + C_p \xi \lambda}{C_r \lambda}} \quad (12)$$

and

$$P_m = \sqrt{\frac{C_t \xi \lambda}{2C_d}} . \quad (13)$$

Note that there are several other solutions. Consequently, we need to verify that the values above produce a cost minima by computing the second derivatives:

$$\begin{aligned} f_{xx} &\triangleq \frac{\partial^2}{\partial N_r^2} \text{Cost}(N_r, P_m) = \frac{4C_d P_m^2 + 2C_p \xi \lambda P_m + 2C_t \xi \lambda}{N_r^3 P_m \lambda} \\ f_{yy} &\triangleq \frac{\partial^2}{\partial P_m^2} \text{Cost}(N_r, P_m) = \frac{4C_d P_m^2 + 2C_p \xi \lambda P_m + 2C_t \xi \lambda}{N_r P_m^3 \lambda} - \frac{8C_d P_m + 2C_p \xi \lambda}{N_r P_m^2 \lambda} + \frac{4C_d}{N_r P_m \lambda} \\ f_{xy} &\triangleq \frac{\partial}{\partial P_m} \frac{\partial}{\partial N_r} \text{Cost}(N_r, P_m) = \frac{2C_d P_m^2 + C_p \xi \lambda P_m + C_t \xi \lambda}{N_r^2 P_m^2 \lambda} - \frac{4C_d P_m + C_p \xi \lambda}{N_r^2 P_m \lambda} . \end{aligned} \quad (14)$$

Clearly,  $f_{xx} > 0$ . From calculus, then, our solutions for  $N_r$  and  $P_m$  will provide minimum cost as long as  $f_{xx} f_{yy} - f_{xy}^2$  is greater than zero. The second derivative test  $f_{xx} f_{yy} - f_{xy}^2$  simplifies to

$$\frac{8C_d^2 C_r^2 \xi \lambda \left( 32C_d C_t^2 + (C_p^3 \xi \lambda + 24C_d C_p C_t) \sqrt{2C_t \xi \lambda / C_d} + 12C_p^2 C_t \xi \lambda \right)}{C_t \left( \sqrt{8C_t C_d \xi \lambda} + C_p \xi \lambda \right)^4} . \quad (15)$$



Moreover, since  $C_t$ ,  $C_r$ ,  $C_p$ ,  $C_d$ ,  $\lambda$ , and  $\xi$  are all positive real numbers,  $f_{xx}f_{yy} - f_{xy}^2$  must be greater than zero. Hence, our equations for  $N_r$  and  $P_m$  describe an optimum cost MIMO search radar.

Next, the cost of this system can be found by substituting our solutions back into the cost function. This cost is

$$Cost(N_r, P_m) = C_f + 2C_r \sqrt{\frac{C_p \xi}{C_r} + \frac{\sqrt{8C_t C_d \xi \lambda}}{C_r \lambda}}. \quad (16)$$

Finally, the optimal values for  $N_t$  and  $L$  can be found by substituting our solutions for  $N_r$  and  $P_m$  into our original constraint equations. The optimal values for  $N_t$  and  $L$  are

$$N_t = \frac{\xi}{\sqrt{\frac{\sqrt{8C_t C_d \xi \lambda} + C_p \xi \lambda}{C_r \lambda}} \cdot \sqrt{\frac{C_t \xi \lambda}{2C_d}}} \quad (17)$$

and

$$L = \frac{\xi \lambda}{\sqrt{2C_t \xi \lambda / C_d}}. \quad (18)$$

Note that the optimal ratio of  $N_r$  to  $N_t$  is

$$\frac{4C_t + C_p \sqrt{2C_t \xi \lambda / C_d}}{2C_r}. \quad (19)$$

What can be said about this ratio? Well, let's start by considering what happens when  $C_t \approx C_r$ . (This case is a reasonable starting point because many of the costs that depend on  $N_t$  and  $N_r$ , such as radiators and structures, will be identical, while other components used to build Rx-only channels will be of similar cost to those used in building the low power parts of the Tx-channels). In this special case, our analysis gives us an important insight:

*With the chosen cost model and constraints, the number of receive elements in an optimal MIMO search radar should be at least double the number of transmit elements.*

These equations also provide another bit of insight. Consider the limit as costs associated with “power density” go to zero (i.e.,  $C_d$  goes to 0). In this limit, it doesn’t cost any more to cool a small array than it does to cool a large one. Moreover, the optimal number of receivers will approach

$$N_r = \sqrt{\frac{C_p \xi}{C_r}} . \quad (20)$$

Likewise, the number of transmitters will go to one (it cannot go below one!). Thus, as  $C_d$  goes to zero, the best MIMO radar is a “ubiquitous radar”—to borrow a term popularized by Merrill Skolnik in [16]. Hence, MIMO radar can be viewed as a generalization of the ubiquitous radar concept, allowing active array antennas with finite cooling costs to provide the benefits of “ubiquitous radar” operation [8].

Before moving to the next section, some final notes. First, the equations above do not explicitly model antenna elevation pattern gain. However, one should be able to simply adjust the PA product to allow for the elevation gain. Second, the above analysis is approximate in the sense that it assumes the dissipated power density is uniform over the full virtual aperture. For MIMO systems employing a large, sparse transmit array (and a small filled receive array), this is approximately true. However, for systems employing a small filled transmit array and a sparse receive array, it is unlikely unless the module power is very low. However, under the cost model above, the use of small filled transmit arrays will be more costly than sparse transmit arrays. Since we seek the minimum cost MIMO system, we need only consider the sparse transmit array case.

### 3.2 OPTIMIZED SEARCH RADAR BASED ON CONVENTIONAL BEAMFORMING

Next, let’s compare the optimized MIMO search radar (above) to a conventional radar system that has been optimized for the same purpose. In the case of the conventional SIMO radar, a single aperture of length  $L$  is assumed to be shared for transmit and receive. Consequently,  $N_t = N_r$  with

$$N_t = 2L/\lambda + 1 \quad N_r = 2L/\lambda + 1 . \quad (21)$$

As above, we constrain search performance via  $P_m \cdot N_t \cdot N_r = \xi$  (or equivalently  $P_m \cdot N_r^2 = \xi$ ). Then, substituting our expression for  $N_t$  into the PA product, we have

$$L = (\lambda/2) \left( \sqrt{\xi/P_m} - 1 \right) . \quad (22)$$

Assuming the cost function has the same form, we substitute our expressions for  $N_t$ ,  $N_r$ , and  $L$  to get

$$Cost(P_m) = \frac{C_r \sqrt{\xi} + C_t \sqrt{\xi} + C_f \sqrt{P_m} + C_p P_m \sqrt{\xi}}{\sqrt{P_m}} - \frac{2C_d P_m \sqrt{\xi}}{\lambda (\sqrt{P_m} - \sqrt{\xi})} \quad (23)$$

and

$$\frac{\partial}{\partial P_m} \text{Cost}(P_m) = \frac{C_p \sqrt{\xi}}{\sqrt{P_m}} + \frac{C_f}{2P_m} - \frac{C_r \sqrt{\xi} + C_t \sqrt{\xi} + C_f \sqrt{P_m} + C_p P_m \sqrt{\xi}}{2P_m^{3/2}} - \frac{2C_d \sqrt{\xi}}{\lambda(\sqrt{P_m} - \sqrt{\xi})} + \frac{C_d \sqrt{P_m} \sqrt{\xi}}{\lambda(\sqrt{P_m} - \sqrt{\xi})^2}. \quad (24)$$

To solve for the minima in closed form, we need to equate this derivative to zero and solve for  $P_m$ , as we did for the MIMO case. In practice, this process is tricky. To approach this problem, we could collect the  $P_m$  terms of  $\partial \text{Cost}(P_m)/\partial P_m$ . This gives a fraction and, since we are setting this fraction equal to zero, we can discard the denominator. The remaining term (i.e., numerator) is a generalized polynomial having components involving  $P_m$  raised to various powers (i.e., 2, 1.5, 1, and 1/2). So, we would next do a change of variables, letting  $x = \sqrt{P_m}$ . This gives us a quartic equation (i.e.,  $x$  is raised to the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> powers). The closed-form solution to such quartic equations, although known, is too long to provide here (Mathematica™ can provide the solution). As a reasonable alternative, one can simply solve numerically for the roots. This is the approach used in Section 3.3 below.

### 3.3 SEARCH RADAR DESIGN EXAMPLE

To illustrate the optimization approaches described in Sections 3.1 and 3.2, we require two sets of input parameters, (1) a radar system design specification (as defined by  $\xi$  and  $\lambda$ ), and (2) a set of cost model coefficients ( $C_t, C_r, C_p, C_d$ , and  $C_f$ ). As a hypothetical example, we assume  $\xi = 1e4$ ,  $\lambda = .02$ ,  $C_t = C_r = 7200$ ,  $C_p = 732$ ,  $C_d = 1200/.2$  (aperture height .2 m), and  $C_f = 2.8e6$ . In addition, 8% of the budget is reserved for test, evaluation, and management. Note that these parameters are meant only to loosely represent costs for a low-production-volume microwave radar system.

Inserting these parameters into our MIMO model (Section 3.1), the optimal number of receive elements (12) is  $N_r = 53.314$ . Likewise, the module power (13) is  $P_m = 10.954$ , the virtual aperture length (18) is  $L = 9.129$ , and the number of transmit elements (17) is  $N_t = 17.122$ . Consequently, the expected cost of the array hardware, as defined by (16), will be \$3.57M excluding overhead. Likewise, the cost including overhead will be \$3.88M.

Of course, we may insert these same parameters and coefficients into the SIMO model of Section 3.2. In doing so, we find that the derivative (24) equals zero at  $P_m = 1.076$ . From (21)–(23), it follows that the optimal SIMO radar is characterized by  $L = 0.954$ ,  $N_r = 96.387$ , and an array cost (including overhead) of \$5.34M.

*Two observations:*

1. The MIMO aperture costs less than the SIMO aperture.
2. The comparison is potentially flawed because it assumes the same cost coefficient values for MIMO and SIMO. This assumption is reasonable only if the SIMO radar uses separate transmit

and receive arrays. In this case, however, there is no reason why the transmit and receive apertures would be equal size. In fact, if separate transmit and receive arrays are used in a SIMO system, many of the same cost advantages will be realized (note: the SIMO system would still have a higher power density and would require more cooling—which is potentially a significant cost factor).

Observation 2 can be addressed via modification of the SIMO cost model. Today, most conventional SIMO radars use the same antennas used for Tx and Rx. In this case, a single module can be used at each antenna location. This module will be more sophisticated than the MIMO transmit (or receive) modules assumed above. For example, it will require a T/R switch or circulator, along with transmit phase and amplitude control. These additional components will increase individual module cost, but this increase will be offset by the need for a single T/R module rather than two separate modules (i.e., Tx-only and Rx-only). In fact, assuming a single T/R module is used instead of two simpler modules, the SIMO cost function will be

$$Cost(P_m) = C_f + \frac{C_t \sqrt{\xi} + C_p P_m \sqrt{\xi}}{\sqrt{P_m}} - \frac{2C_d P_m \sqrt{\xi}}{\lambda(\sqrt{P_m} - \sqrt{\xi})} . \quad (25)$$

Furthermore, allowing the T/R module cost parameter to rise to  $C_t = 8900$ , the partial derivative of Equation (25) will equal zero at  $P_m = 0.777$  W. From Equations (21), (22), and (25), this implies the optimal SIMO radar will be characterized by  $L = 1.124$ ,  $N_r = 113.446$ , and a resulting array cost, including overhead, of \$4.72M.

Note that, even with the sharing of transmit and receive hardware, conventional radar costs are expected to significantly exceed MIMO radar costs.



## 4. APERTURE OPTIMIZATION WITH RESOLUTION CONSTRAINT

In this section, we address the problem of designing optimal search radars providing a specified spatial resolution. As in the previous section, we consider both MIMO and SIMO systems.

### 4.1 OPTIMIZED MIMO SEARCH RADAR WITH RESOLUTION CONSTRAINT

First, we consider the design of MIMO radars that minimize cost under both power-aperture and resolution constraints. As in our earlier MIMO discussion, a critically sampled MIMO virtual array is assumed. Hence,  $N_t$ ,  $P_m$ , and the cost function are as defined in Section 3.1. This time, however, we begin by substituting our expressions for  $N_t$  and  $P_m$  into our cost function, yielding

$$Cost(N_r) = C_r N_r + C_f + \frac{C_p \xi}{N_r} + \frac{C_d \xi}{N_r L} + \frac{2C_t L}{N_r \lambda} . \quad (26)$$

Next, since resolution is constrained (i.e.,  $L$  is given), we simply choose  $N_r$  to minimize cost. The derivative of our cost function with respect to  $N_r$  is

$$\frac{\partial}{\partial N_r} Cost(N_r) = C_r - \frac{2C_t L^2 + C_p \xi \lambda L + C_d \xi \lambda}{N_r^2 \lambda L} . \quad (27)$$

The derivative equals zero when

$$N_r = \sqrt{\frac{2C_t L^2 + C_p \xi \lambda L + C_d \xi \lambda}{C_r L \lambda}} . \quad (28)$$

Likewise, the second derivative of our cost function is

$$f_{xx}(N_r) = \frac{\partial^2}{\partial N_r^2} Cost(N_r) = \frac{2C_d \xi + 2C_p L \xi}{N_r^3 L} + \frac{4C_t L}{N_r^3 \lambda} . \quad (29)$$

At the critical point of Equation (28), it can be shown that  $f_{xx}(N_r)$  is positive. Thus, the critical point gives the value of  $N_r$  associated with the minimum cost design solution. Likewise, substituting Equation (28) into Equation (4) yields the optimal  $N_t$ , with  $P_m$  computed directly from Equation (6).

Note that the optimal ratio  $N_r/N_t$  is now given by

$$\frac{C_t}{C_r} + \frac{C_d \xi \lambda + C_p L \xi \lambda}{2C_r L^2} . \quad (30)$$

Furthermore, this ratio is directly related to the optimal unconstrained MIMO system through Equation (18), i.e., substituting Equation (18) into Equation (30) yields Equation (19).

Lastly, observe that as  $L$  grows larger in Equation (30), the costs associated with cooling become less significant. Likewise, the module power decreases (to a point which it is dominated by the load from the low-power electronics), reducing costs associated with high-power amplification, DC conversion, power conditioning, and chilling. In the limit, as  $L$  grows very large, the  $C_t$  and  $C_r$  cost terms dominate the optimization. Furthermore, if  $C_t \approx C_r$ , then (in the limit) the optimum system will have  $N_t \approx N_r$ . In this sense, the system will be “balanced.” However, the aperture size required to approach this limit can be quite large, making this limiting scenario the exception, not the rule.

## 4.2 OPTIMIZED SIMO SEARCH RADAR WITH RESOLUTION CONSTRAINT

Finally, let’s compare our optimized MIMO search radar, with resolution constraints, to a conventional SIMO system designed for the same purpose. Here,  $N_t$  and  $N_r$  are defined by Equation (21) and the power-aperture product is constrained via Equation (5). With the resolution also constrained (i.e.,  $L$  given), the only unknown is  $P_m$ . Thus, by substituting Equation (21) into Equation (5), we have

$$P_m = \xi \lambda^2 / (2L + \lambda)^2. \quad (31)$$

The cost is then computed directly from Equation (25) or (23), depending upon whether a single aperture (i.e., shared transmit and receive array) or pair of apertures is used.

## 4.3 RESOLUTION CONSTRAINED SEARCH RADAR EXAMPLE

To illustrate the incorporation of resolution constraints, let’s assume the same values for  $C_t$ ,  $C_p$ ,  $C_d$ , and  $C_r$  as in Section 3.1. Furthermore, suppose we constrain  $L = 25$  meters (the approximate useable length of a narrow-body airplane, such as 737, A-318, or E170). In this case, Equation (28) dictates that the optimum MIMO radar system will have  $N_r = 62.048$ . This implies  $N_t = 40.291$ ,  $P_m = 4$ , and the projected array cost (including overhead) for this system will be \$4.02M.

Likewise, assuming the same values for  $C_t$ ,  $C_p$ ,  $C_d$ ,  $C_r$ , and  $L$ , Equation (21) dictates the optimum SIMO radar will have  $N_t = N_r = 2501$ . Furthermore, Equation (31) dictates  $P_m = 1.599\text{e-}3$ , which results in a projected array cost (including overhead) of \$42.19M.

Of course, as discussed in Section 3.2, it is common to implement SIMO radars using a shared T/R aperture and combined T/R modules (rather than separate Tx and Rx modules). In this case, the optimum SIMO radar will still have  $N_t = N_r = 2501$  and  $P_m = 1.599\text{e-}3$ . Moreover, the projected array cost (including overhead) drops to \$27.24M because of efficiencies gained by using combined T/R modules and a shared T/R aperture. However, the SIMO radar cost is still nearly an order of magnitude greater than the MIMO system cost, while providing the same resolution and PA product. This higher cost is partially because we required an element digital SIMO array architecture. Today, large SIMO radars usually employ analog subarray beamforming. Using analog subarrays will further reduce the SIMO radar’s cost.

## 5. WEIGHT OPTIMIZATION

In Sections 3 and 4, our analysis focused explicitly on minimizing aperture cost. Clearly, cost minimization is not always the designer's main objective. For example, systems that will fly on small aircraft may be driven by the need to minimize payload weight rather than cost. Nonetheless, the first order objective function will still take the form given by Equation (8). Hence, even though the objective function coefficient values will change, our optimization approach, as well as the equations describing the solution, should still apply.

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## 6. SUMMARY AND CONCLUSIONS

Applying traditional engineering techniques, it has been shown that it is possible to optimize MIMO and SIMO apertures to minimize an objective (e.g., cost or weight function) subject to constraints on performance. A simple procedure was presented. Although simplistic, our procedure provided useful insight into the amount of savings that can be achieved by using MIMO technology. It also provided insights into various MIMO radar design relationships. For example, under the assumed hardware and cost model (for active element-digital arrays), MIMO search apertures typically have  $N_r \gg N_t$ .

To illustrate our approach, we presented design examples assuming  $\xi = 10000$ , as well as various hypothetical cost parameters (meant to represent a low-production microwave radar). Table 2 summarizes the resulting system designs.

**Table 2**  
**Summary of SIMO and MIMO Radar Design Examples**

	CONSTRAINTS	NO. OF TRANSMITTERS	NO. OF RECEIVERS	LENGTH (REAL/VIRTUAL)	MODULE POWER	APERTURE COST (\$)	Note
SIMO	PA	96	96	0.95 m	1.1 Watts	\$5.3M	(1)
		113	113	1.1 m	.8 Watts	\$4.7M	(2)
MIMO	PA	17	53	9.1 m	10.9 Watts	\$3.9M	(1)
SIMO	PA & Resolution	2501	2501	25 m	1.6 mWatts	\$42.2M	(1)
		2501	2501	25 m	1.6 mWatts	\$27.2M	(2)
MIMO	PA & Resolution	42	62	25 m	4 Watts	\$4.0M	(1)

(1) Separate T and R modules and antennas

(2) Combined T/R module and antenna

Our example suggests that a power-aperture (PA) constrained MIMO aperture will be moderately less expensive as compared with a comparable SIMO aperture. In contrast, when both power-aperture and resolution are constrained, MIMO apertures can be dramatically less costly. It should be noted, however, that MIMO systems require additional digital processing (not considered here). MIMO radars may also sacrifice some flexibility, which is important in applications requiring multimode or multifunction operation. Furthermore, "critically sampled" MIMO radars will be more degraded by module failures than comparable SIMO active electronically scanned array (AESA) radars. As a result, additional cost may be incurred to incorporate fault tolerance. Finally, limited waveform orthogonality is known to degrade MIMO radar system performance in a way that is unique to MIMO radars. Hence, cost is only one factor that should be considered in selection of a radar architecture.

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